Chapter 9

Sinusoidal Steady-State Analysis

So far we have discussed circuits with constant sources. In this chapter we will consider circuits driven by time-varying current or voltage sources.

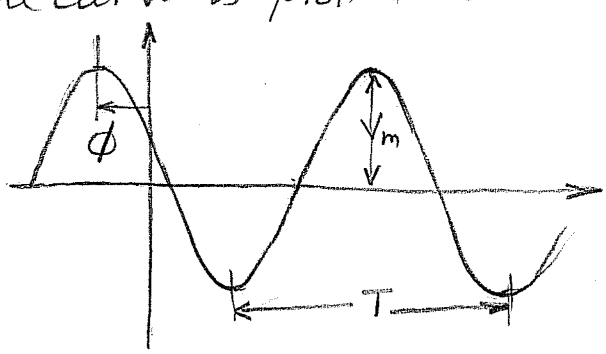
We are interested in sources with voltages and currents varying sinusoidally in time because such sources are used in the generation, transmission, and distribution of electric power.

9.1 The Sinusoidal Source

A sinusoidal voltage source produces a voltage that varies sinusoidally with time. The sinusoidal voltage source is expressed as:

$$v = V_m \cos(\omega t + \phi) \quad V$$

The curve is plotted as:



The sinusoidal function repeats itself at regular intervals. Each interval is called a period and denoted by T. The reciprocal of T gives the number of cycles per second or frequency:

$$f = 1/T \quad \text{(hertz) or (Hz)}$$

The coefficient of t is the angular frequency:

$$\omega = 2\pi f \qquad rad/s$$

The coefficient V_m is the amplitude of the voltage.
The angle ϕ is the phase angle of the sinusoidal voltage. It determines the value of the voltage at $t=0$. ϕ is expressed in radians.

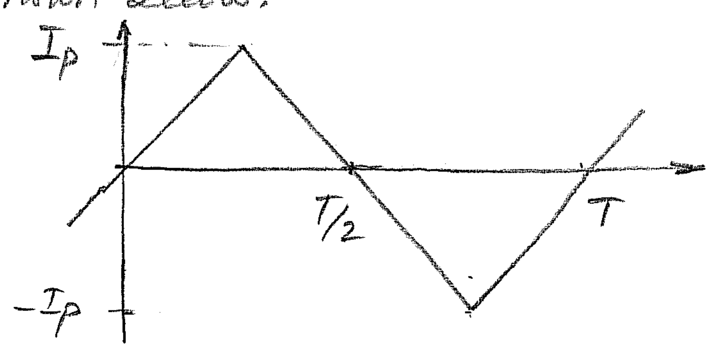
Another important characteristic of the voltage is its root-mean-square (rms) value defined by:

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t+T} V_m^2 \cos^2(\omega t+\phi)\,dt} \Rightarrow$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Example 9.4: The rms of triangular waveform

Calculate the rms of a triangular wave current shown below:



From the definition:

$$I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2\,dt}$$

The analytical expression of i is:

$$i(t) = \frac{4I_p}{T} t \qquad 0 \le t \le T/4$$

So

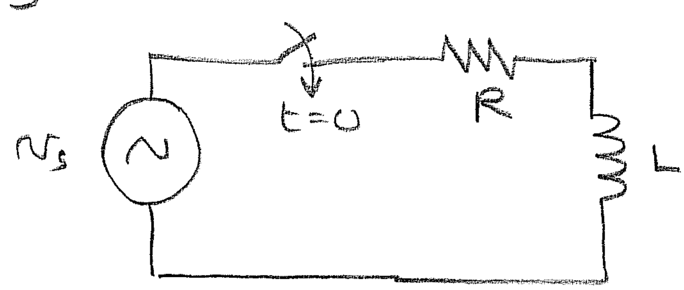
$$\int_{t_0}^{t_0+T} i^2 dt = 4 \int_0^{T/4} \frac{16 I_p^2}{T^2} t^2 dt = \frac{I_p^2 T}{3}$$

Hence

$$I_{rms} = \frac{I_p}{\sqrt{3}}$$

9.2 The Sinusoidal Response

The general response to sinusoidal sources may be introduced using the following circuit:



$$v_s = V_m \cos(\omega t + \phi)$$

The Differential equation describing the circuit is obtained using KVL:

$$L \frac{di}{dt} + Ri = V_m \cos(\omega t + \phi) \qquad 9.8$$

The solution to the above DE is given by:

$$i(t) = -I_m \cos(\phi - \theta) e^{-t/\tau} + I_m \cos(\omega t + \phi - \theta) \quad 9.9$$

where:

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}, \quad \tau = \frac{L}{R}, \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The first term on the rhs of (9.9) is the <u>transient</u> component which decays as t increases. The second term is the <u>steady-state</u> component of the solution.

Let us focus on the steady state component:

1. It is a sinusoidal function.
2. The frequency of the response is identical to that of the source
3. The maximum amplitude of the current response is given as $I_m = V_m / \sqrt{R^2 + \omega^2 L^2}$
4. The phase angle of the current response is different from the phase angle of the source.

9.3 The Phasor

The phasor is a complex number that carries the amplitude and phase angle information. The phasor concept is rooted in Euler's identity:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

So

$$\cos\theta = \mathcal{R}\{e^{j\theta}\} \qquad (9.11)$$

and $\sin\theta = \Im m\{e^{j\theta}\}$

If we apply 9.11 to the sinusoidal voltage:

$$v = V_m \cos(\omega t + \phi)$$
$$= V_m \mathcal{R}\{e^{j(\omega t+\phi)}\} = V_m \mathcal{R}\{e^{j\phi} e^{j\omega t}\}$$

The quantity $V_m e^{j\phi}$ carries the amplitude and phase angle of the given sinusoidal voltage. This is key definition the phasor representation of the sinusoidal signal. It is obtained using the phasor transform $\mathcal{P}$:

$$V = V_m e^{j\phi} = \mathcal{P}\{V_m \cos(\omega t + \phi)\} \qquad (9.15)$$

The phasor transform changes the time function into a complex number, which is also called the frequency domain.

Equation 9.15 is the polar form of the phasor which may abbreviated using the angle notation:

$$V = V_m \angle\phi$$

The rectangular form of the phasor is:

$$V = V_m \cos\phi + j V_m \sin\phi \qquad (9.16)$$

The Inverse Phasor Transform

$$\mathcal{P}^{-1}\{V_m e^{j\phi}\} = \mathcal{R}\{V_m e^{j\phi} e^{j\omega t}\}$$
$$= V_m \cos(\omega t + \phi).$$

The phasor of the current response (steady state) is:

$$I = \mathcal{P}\{I_m \cos(\omega t + \phi - \theta)\}$$
$$= I_m e^{j(\phi-\theta)} = I_m e^{j\beta}$$

But $\qquad I_m = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$

So $\qquad I = \dfrac{V_m e^{j\phi}}{\sqrt{R^2 + \omega^2 L^2}\, e^{j\theta}}$

The denominator is the polar form of the complex number $Z = R + j\omega$ known as the impedance:

$$I = \frac{V_m e^{j\phi}}{Z e^{j\theta}} = \frac{V_m e^{j\phi}}{R + j\omega L} \qquad (9.22)$$

Example 9.5: Adding cosine functions using Phasors.

Let $y_1 = 20\cos(\omega t - 30^\circ)$ and

$y_2 = 40\cos(\omega t + 60^\circ)$

express $y = y_1 + y_2$ as a single sinusoidal function!

$$y = y_1 + y_2 \Rightarrow Y = Y_1 + Y_2$$

So
$$Y = 20\angle{-30^\circ} + 40\angle{60^\circ}$$
$$= (17.32 - j10) + (20 + j34.64)$$
$$= 37.32 + j24.64 = 44.72\angle{33.43^\circ}$$

$$y = \mathcal{P}^{-1}\{Y\} = \mathcal{P}^{-1}\{44.72\angle{33.43}\} \Rightarrow$$

$$y = 44.72\cos(\omega t + 33.43^\circ)$$

Assessment Problem 9.1

Find the phasor of each trigonometric function:

a) $v = 170\cos(377t - 40^\circ)$ V

$$V = \mathcal{P}\{v\} = 170\,e^{-j40^\circ} = 170\angle{-40^\circ} \text{ V}$$

b) $i = 10\sin(1000t + 20)$ A

$$I = \mathcal{P}\{i\} = 10\angle{20^\circ} \text{ A}$$

c) $i = 5\cos(\omega t + 36.87^\circ) + 10\cos(\omega t - 53.13^\circ)$ A.

$I = \mathcal{P}\{i\} = 5\angle 36.87 + 10\angle -53.13$

$= 4 + j3 + 6 - j8 = 10 - j5$

$= 11.18\angle -26.57^\circ$ A.

A.P. 9.2

Find the time domain expression corresponding to each phasor

a) $V = 18.6\angle -54^\circ$ V

$v = \mathcal{P}^{-1}\{V\} = 18.6\cos(\omega t - 54^\circ)$ V

b) $I = 20\angle 45^\circ - 50\angle -30^\circ$ mA.

$= 14.142 + j14.42 - 43.30 + j25$ mA.

$= -29.16 + j39.14 = 48.81\angle 126.68^\circ$ mA

$i = \mathcal{P}^{-1}\{I\} = 48.81\cos(\omega t + 126.68^\circ)$ mA.

c) $V = (20 + j80) - 30\angle 15$ V

$= (20 + j80) - (28.98 + j7.765)$

$= -8.98 + j72.24$

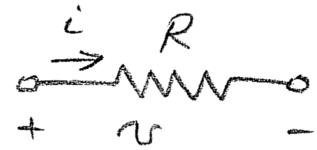
$= 72.79\angle 97.08^\circ \Rightarrow$

$v = \mathcal{P}^{-1}\{V\} = 72.79\cos(\omega t + 97.08^\circ)$ V

9.4 The Passive Circuit Elements in the Frequency Domain

The V-I Relationship for a Resistor

For a resistor with current i as shown, the voltage v is given by Ohm's law:



$$v = Ri$$

So if $i = I_m \cos(\omega t + \theta_i)$

then $$v = R I_m \cos(\omega t + \theta_i) \qquad (9.25)$$

which may be written as:

$$v = V_m \cos(\omega t + \theta_i) \qquad (V_m = R I_m).$$

Equation 9.25 in the phasor domain becomes:

$$V = R I_m \angle \theta_i$$

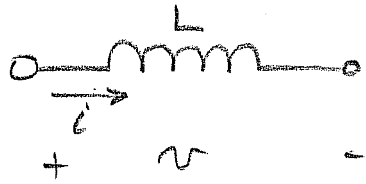
But $I_m \angle \theta_i$ is the phasor representation I of the sinusoidal current. So

$$V = RI \qquad (9.27)$$

The phasor voltage is the phasor current times the resistance.

The V-I Relation for an Inductor

Consider an inductor L with current i:



Let $i = I_m \cos(\omega t + \theta_i)$

The voltage is given by:

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \theta_i) \Rightarrow$$

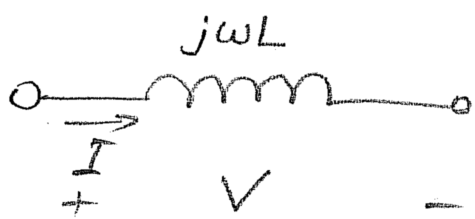
$$v = -\omega L I_m \cos(\omega t + \theta_i - 90^\circ)$$

The phasor representation of v is:

$$V = \mathcal{P}\{v\} = -\omega L I_m e^{j(\theta_i - 90^\circ)}$$

$$= -\omega L I_m e^{j\theta_i} e^{-j90^\circ} \qquad (e^{-j90^\circ} = -j\,!)$$

So $\quad V = j\omega L\, I \qquad\qquad (9.30)$



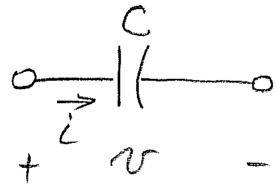
Another way of writing equation 9.30 is:

$$V = j\omega L\, I = (\omega L \angle 90^\circ)(I_m \angle \theta_i) \Rightarrow$$

$$V = \omega L I_m \underline{\angle \theta_i + 90^\circ}$$

The V-I Relation for a Capacitor

Consider a capacitor C with voltage v:



$$v = V_m \cos(\omega t + \theta_v)$$

The current i through the capacitor is:

$$i = C\frac{dv}{dt}$$

Then $\quad I = j\omega C V \Rightarrow \qquad (9.32)$

$$V = \frac{1}{j\omega C} I \qquad (9.33)$$

Equation 9.33 can be expressed as:

$$V = \left(\frac{1}{\omega C} \angle -90^\circ\right) \times \left(I_m \angle \theta_i\right) \Rightarrow$$

$$V = \frac{I_m}{\omega C} \angle \theta_i - 90^\circ \qquad (9.34)$$

We conclude:

$$V = ZI$$

with $Z = R$ for a resistor R

$Z = j\omega L$ for a inductor L

$Z = \frac{1}{j\omega C}$ for a capacitor C.

9.5 Kirchoff Voltage & Current Laws in the Phasor Domain

Consider a loop in a circuit with voltages $v_1, v_2, \ldots v_n$. KVL requires that:

$$v_1 + v_2 + \cdots + v_n = 0$$

If the voltages are sinusoidal state:

$$V_{m1}\cos(\omega t+\theta_1) + V_{m2}\cos(\omega t+\theta_2) + \cdots + V_{mn}\cos(\omega t+\theta_n) = 0$$

Using Euler's identity the above equation can be written as:

$$\mathcal{R}\{V_{m1}e^{j\omega t}e^{j\theta_1} + \cdots + V_{mn}e^{j\omega t}e^{j\theta_n}\} = 0 \Rightarrow$$

$$\mathcal{R}\{(V_{m1}e^{j\theta_1} + \cdots + V_{mn}e^{j\theta_n})e^{j\omega t}\} = 0 \Rightarrow$$

$$\mathcal{R}\{(V_1 + V_2 + \cdots + V_n)e^{j\omega t}\} = 0$$

since $e^{j\omega t} \neq 0$ so

$$\underline{V_1 + V_2 + \cdots + V_n = 0} \qquad (9.41)$$

Now consider a node with currents $i_1, i_2 \ldots i_n$. KCL requires that:

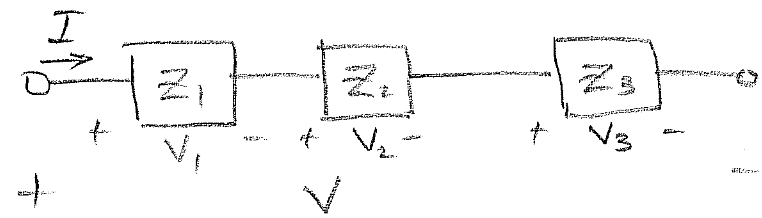
$$i_1 + i_2 + \cdots + i_n = 0$$

then

$$\underline{I_1 + I_2 + \cdots I_n = 0} \qquad 9.43$$

9.6 Series, Parallel, and Δ-Y Simplifications

Impedances in Series



From KVL: $V = V_1 + V_2 + V_3 \Rightarrow$

By Ohm's Law: $V = Z_1 I + Z_2 I + Z_3 I \Rightarrow$

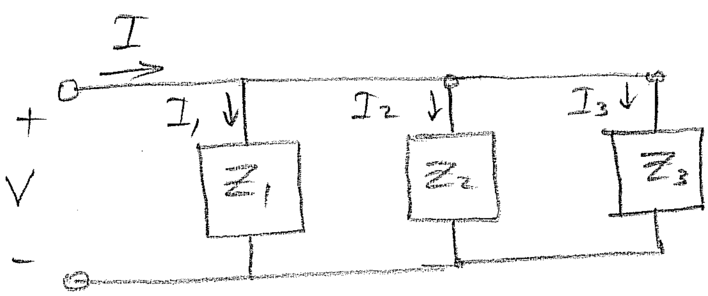
$$V = (Z_1 + Z_2 + Z_3) I \Rightarrow$$

$$V = Z_{eq} I$$

with $Z_{eq} = Z_1 + Z_2 + Z_3$

So impedances in series add.

Impedances in Parallel



KCL: $I = I_1 + I_2 + I_3 \Rightarrow$

Ohm's Law: $I = \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3} \Rightarrow$

$$I = \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right) V \Rightarrow$$

$$I = \frac{V}{Z_{eq}}$$

$$\frac{1}{Z_{eq}} = \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)$$

For the special case of two impedances:

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

The inverse of an impedance Z is the admittance:

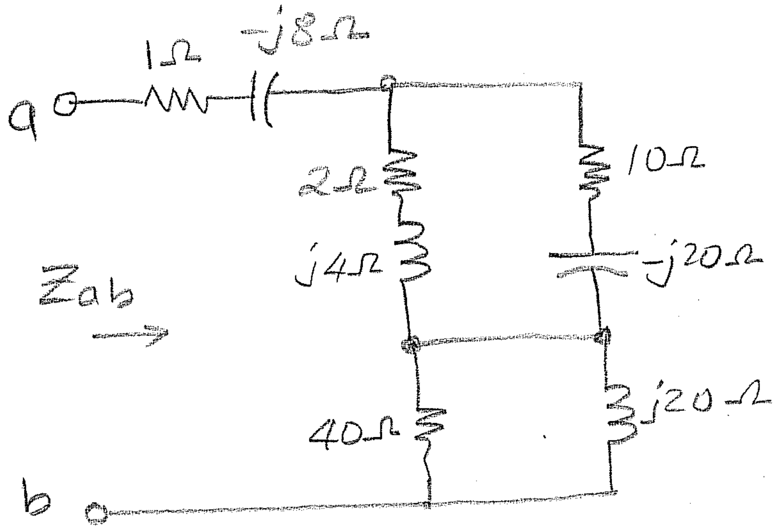
$$Y = \frac{1}{Z} = \frac{1}{R + jX} = G + jB \quad S \text{ (siemens)}$$

G is called the conductance and B is called the susceptance.

Study Example 9.7 and Solve Assessment Problems 9.7 and 9.8.

Problem 9.27

Find Z_{ab} for the circuit shown. Express it in cartesian and Polar form.



Let $Z_1 = 1 - j8\ \Omega$

$Z_2 = 2 + j4\ \Omega$ $\quad Z_3 = 10 - j20\ \Omega$

$Z_4 = 40\ \Omega$ $\quad Z_5 = j20\ \Omega$

Z_{ab} can written as:

$$Z_{ab} = Z_1 + (Z_2 \| Z_3) + (Z_4 \| Z_5)$$

$$Z_2 \| Z_3 = \left(\frac{1}{Z_2} + \frac{1}{Z_3}\right)^{-1} = \left(\frac{1}{2+j4} + \frac{1}{10-j20}\right)^{-1}$$

$$= (0.12 - j0.16)^{-1} = 3 + j4 \ \Omega$$

$$Z_4 \| Z_5 = \left(\frac{1}{Z_5} + \frac{1}{Z_4}\right)^{-1} = \left(\frac{1}{40} + \frac{1}{j20}\right)^{-1}$$

$$= (0.025 - j0.05)^{-1} = 8 + j16 \ \Omega$$

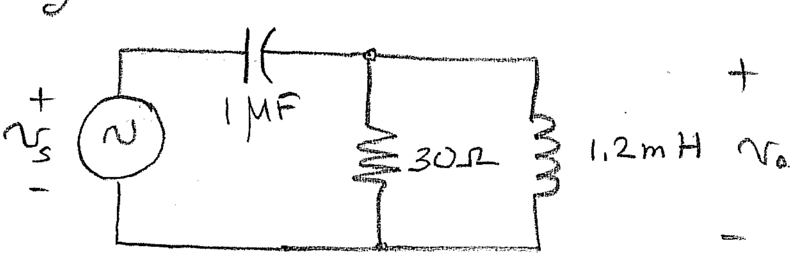
So

$$Z_{ab} = 1 - j8 + 3 + j4 + 8 + j16 \Rightarrow$$

$$Z_{ab} = 12 + j12 \ \Omega$$

Problem 9.29

Find the steady state value of $v_o(t)$ in the following circuit:



with $v_s = 40 \cos 50\,000t$ V.

Solution:

The angular frequency $\omega = 50\,000$ rad/s
The impedance associated with $C = 1\,\mu F$ is:

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j50000 \times 1 \times 10^{-6}} = \frac{1}{j0.05} = -j20\ \Omega$$

$$Z_R = R = 30\Omega \quad \text{and}$$

$$Z_L = j\omega L = j\,50\,000 \times 1.2 \times 10^{-3} = j60\,\Omega$$

Since KVL and KCL apply, then the voltage divider rule also applies:

$$V_0 = V_S \frac{(Z_R // Z_L)}{Z_C + (Z_R // Z_L)}$$

$$Z_R // Z_L = \left(\frac{1}{30} + \frac{1}{j60}\right)^{-1} = (0.0333 + j0.0167)^{-1}$$

$$= 24 + j12\ \Omega$$

$V_S = 40$ V (Note $\theta_v = 0 \Rightarrow e^{j\theta_v} = 1$).

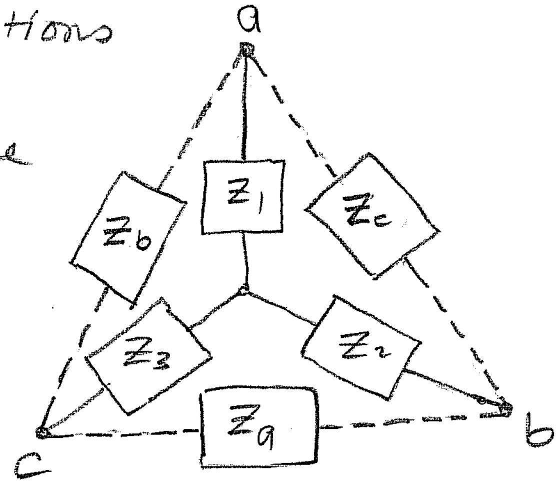
So

$$V_0 = 40 \frac{24 + j12}{-j20 + 24 + j12} = 40 \times (0.75 + j0.75)$$

$$= 30 + j30 = 42.426 \angle 45^\circ$$

So $v_0(t) = 42.426 \cos(50000t + 45^\circ)$ V

Delta to Wye Transformations



The Δ-Y transformations are

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} \quad \text{and}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

The Y-Δ Transformations are:

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

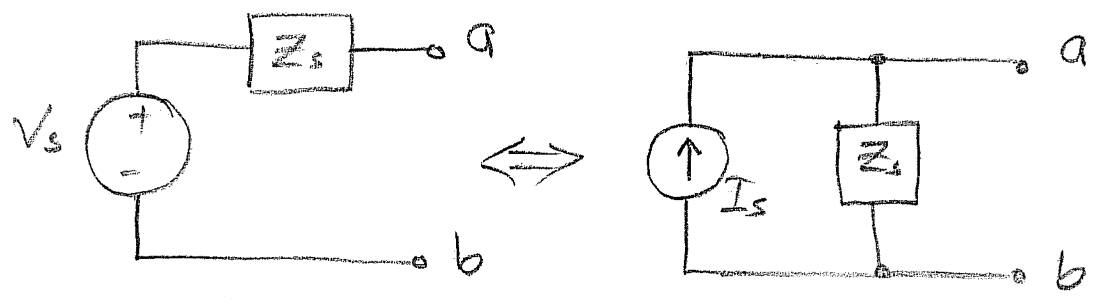
$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2} \quad \text{and}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

Study Example 9.8 and solve 9.37.

9.7 Source Transformations and Thévenin-Norton Equivalent Circuits

Source Transformation in the phasor domain:



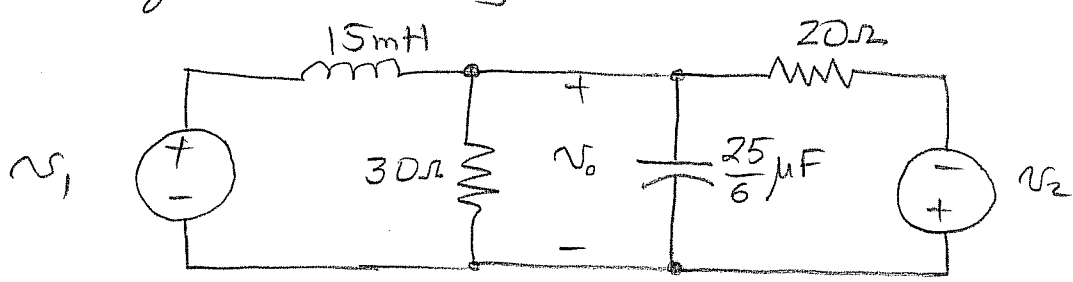
$V_s = Z_s I_s$ $\qquad$ $I_s = \dfrac{V_s}{Z_s}$

Also a frequency-domain linear circuit with independent and dependent sources may be represented by its Norton or Thevenin equivalent circuit.

Study examples 9.9 and 9.10.

Assessment Problem 9.10 on Source Transformation.

For the circuit shown, find the steady-state voltage $v_o(t)$ using source transformation:



with $v_1 = 240 \cos(4000t + 53.13)$ V

$v_2 = 96 \sin 4000t$ V

$V_1 = 240\,\angle 53.13^\circ$ and $V_2 = 96\,\angle -90^\circ$ V

$\sin\theta = \cos(\theta - 90^\circ)$!

$Z_L = j\omega L = j\,4000 \times 15 \times 10^{-3} = j\,60\,\Omega$

$Z_C = \dfrac{1}{j\omega C} = \dfrac{1}{j\,4000 \times \frac{25}{6} \times 10^{-6}} = \dfrac{1}{j0.0167} = -j\,60\,\Omega$

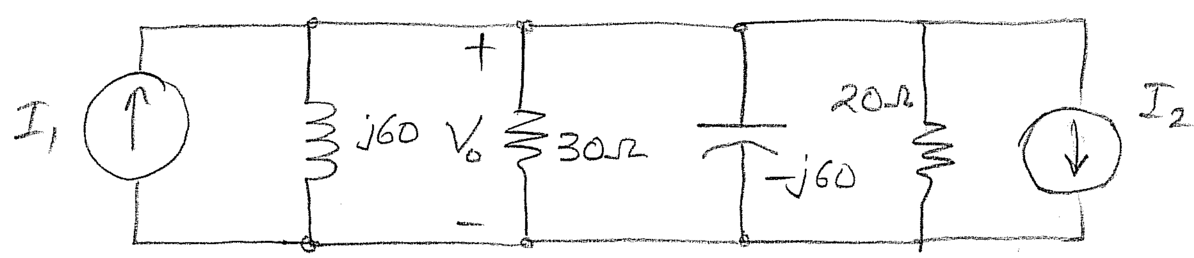
The current source associated with V_1 is:

$$I_1 = \frac{240\,\angle 53.13}{j60} = 3.2 - j2.4 = 4\,\angle -36.87^\circ \text{ A}$$

The current source associated with V_2 is:

$$I_2 = \frac{96\,\angle -90^\circ}{20} = 4.8\,\angle -90^\circ \text{ A}$$

The equivalent circuit with current sources:

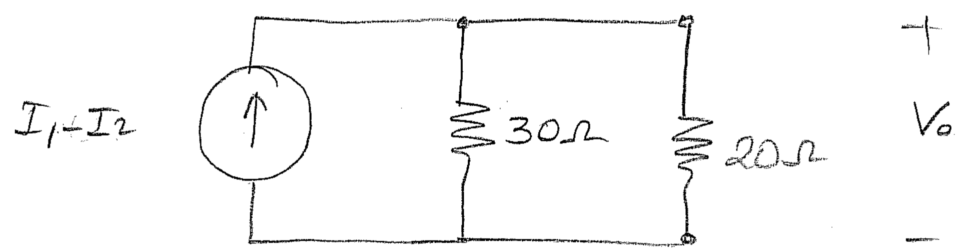


The inductive and capacitive impedances, Z_L and Z_c respectively appear in parallel:

$$\frac{1}{Z_{eq}} = \frac{1}{Z_L} + \frac{1}{Z_c} = \left(\frac{1}{j60} - \frac{1}{j60}\right) = 0$$

So $Z_{eq} \to \infty \Rightarrow$ open circuit!

So the equivalent circuit becomes:



$V_o = (I_1 - I_2)\, R_p$

$I_1 - I_2 = 4\angle{-36.87^\circ} - 4.8\angle{-90^\circ}$

$=$

$R_p = \dfrac{20 \times 30}{20 + 30} = 12\ \Omega$

$V_o = (3.2 + j2.4) \times 12 = 38.4 + j28.8\ \text{V}$

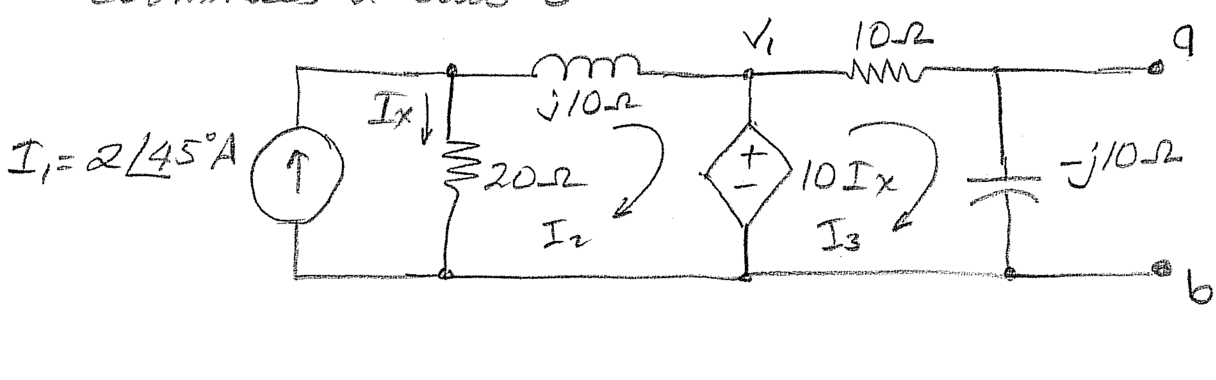
$= 48\angle{36.87^\circ}\ \text{V}$

So

$v_o = 48 \cos(4000t + 36.87^\circ)\ \text{V}.$

Assessment Problem 9.11

Find the Thevenin equivalent with respect to terminals a and b:



The open-circuit voltage Voc is given by:

$$V_{oc} = -j10\, I_3$$

The mesh current I_3 can be found from the mesh current equations:

Loop 2: $\quad j10 I_2 + 10x - 20 I_x = 0 \qquad (1)$

Loop 3: $\quad 10 I_x = (10 - j10) I_3 \qquad (2)$

From KCL at current source node:

$$I_2 = I_1 - I_x \qquad (3)$$

So we have 3 equations in 3 unknowns. But we only need to get I_3. So use (3) in (1) to get a value for I_x which then can be used in (2) to get I_3:

(3) and (1) $\Rightarrow$ $j10(I_1 - I_x) - 10 I_x = 0 \Rightarrow$

$$I_x = \frac{j}{1+j} I_1 = \frac{j}{1+j}\, 2\angle 45^\circ = j\sqrt{2}$$

Replace in (2) $\Rightarrow$ $10 j\sqrt{2} = 10(1-j)\, I_3 \Rightarrow$

$$I_3 = \frac{j\sqrt{2}}{1-j} = 1\angle 135^\circ \text{ A}$$

$$V_{oc} = -j10\, I_3 = -j10 \times (1\angle 135)$$

$$V_{Th} = 10\angle 45^\circ \text{ V}$$

The Thevenin Impedance is deduced from the short circuit current. When a and b are shorted the $-j10\Omega$ capacitance is shorted.

The loop 2 equation remains the same:

$$j10 I_2 - 10 I_x = 0 \qquad (4)$$

The loop 3 equation becomes:

$$10 I_x = 10 I_3 \qquad (5)$$

And KCL at source node:

$$I_2 = I_1 - I_x$$

Hence $I_x = j\sqrt{2}$

Replace in 5: $I_3 = I_x = j\sqrt{2}$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{10\angle 45}{j\sqrt{2}} = 5 - j5\ \Omega.$$

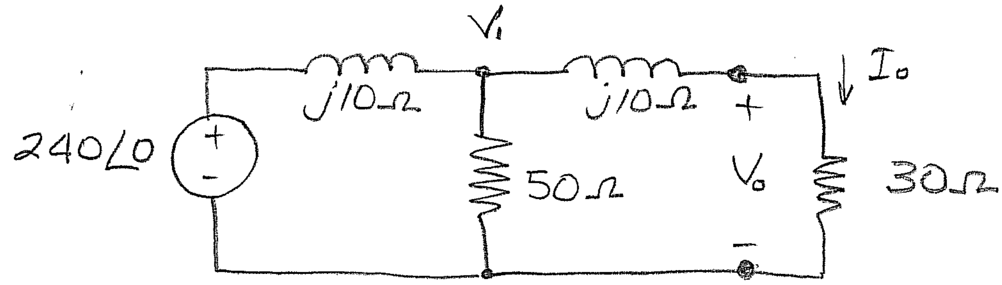
9.8 The Node Voltage Method

The same concepts apply to the NVM in the phasor (or frequency) domain as the NVM for dc sources.

Study Example 9.11 and solve Assessment Problem 9.11

Problem 9.55

Use the NVM to find V_o in the following circuit:



The circuit has two essential nodes one of which is set as a common node and the other is tagged with V_1. Once V_1 is found using the NVM V_o is found using the voltage divider rule:

$$V_o = V_1 \frac{30}{30+j10} \quad V$$

KCL at node 1 expressed in terms of V_1:

$$\frac{V_1 - 240}{j10} + \frac{V_1}{50} + \frac{V_1}{30+j10} = 0$$

$$V_1\left(\frac{1}{j10}+\frac{1}{50}+\frac{1}{30+j10}\right)=\frac{240}{j10} \Rightarrow$$

$$V_1\left(0.05-j0.11\right)=\frac{240}{j10} \Rightarrow$$

$$V_1=\frac{240}{(j10)(0.05-j0.11)}=180.8-j\,82.2 \text{ V}$$

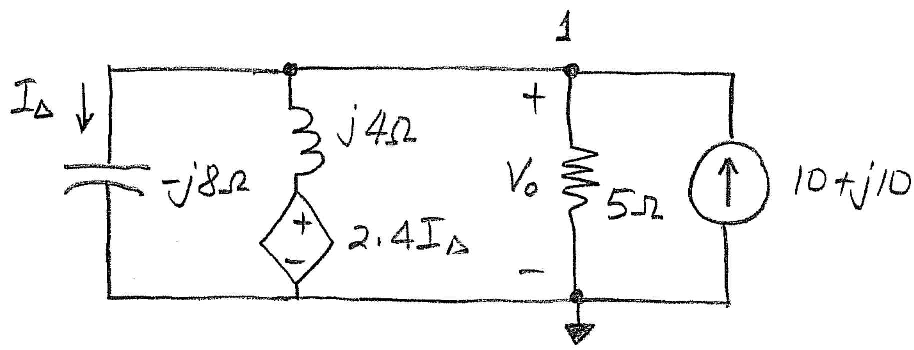
$$=198.6\angle{-24.4^\circ} \text{ V}$$

$$V_o=198.6\angle{-24.4^\circ}\times\frac{30}{30+j10}=138.1-j\,128.2$$

$$=188.43\angle{-42.88^\circ} \text{ V}$$

Problem 9.59

Use the NVM to find the phasor voltage V_o in the following circuit.



Again the circuit here has two essential nodes.

KCL at node 1 in terms of V_o:

$$\frac{V_o}{5}+\frac{V_o-2.4I_\Delta}{j4}+\frac{V_o}{-j8}=10+j10$$

but $I_\Delta=\frac{V_o}{-j8}$, so replace in KCL to obtain:

$$V_o\left(\frac{1}{5}+\frac{1}{j4}+\frac{2.4}{j4}\times\frac{1}{j8}-\frac{1}{j8}\right)=10+j10$$

$$V_o=\frac{10+j10}{(0.125-j0.125)}=j80=80\underline{/-90^\circ}\ V$$

Solve also 9.56 and 9.57!

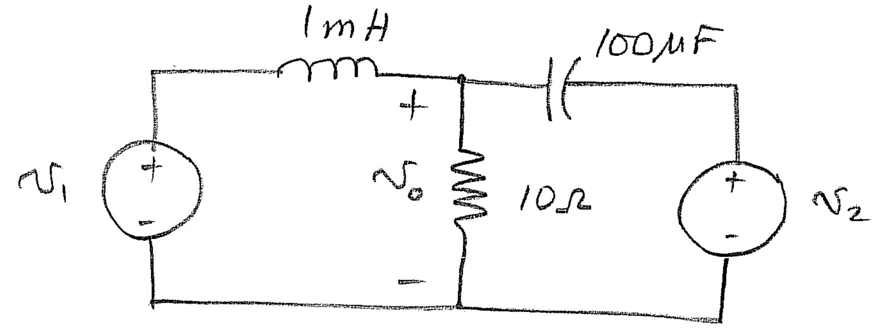
9.9 The Mesh-Current Method.

Here also the same principles used in dc current analysis for the MCM apply in the phasor domain.

Study Example 9.12 and Solve Assessment Problem 9.13 using the MCM. Try solving 9.13 using the NVM. Which is easier to apply to this problem?

In what follows I will solve 9.60 and 9.64.

9.60 Use the MCM to find the steady state expression for v_o in the following circuit:



$v_1 = 20 \cos(2000t - 36.87°)$ V

$v_2 = 50 \sin(2000t - 16.26°)$ V

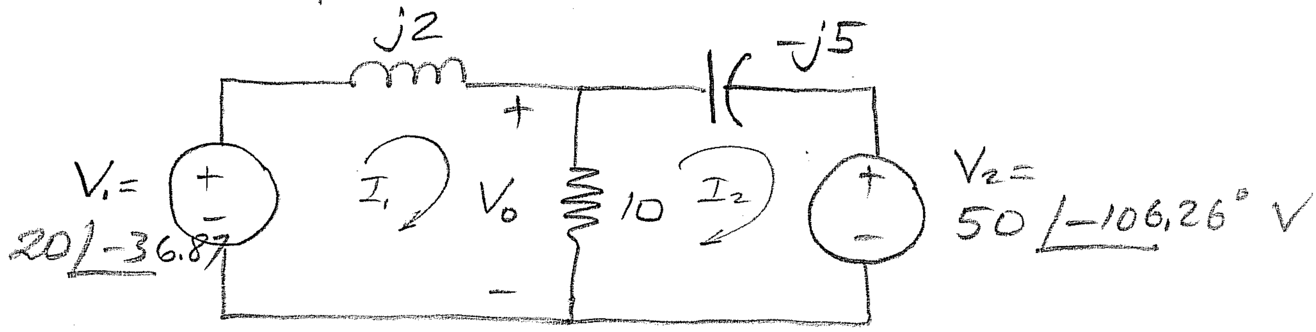
First convert all sources and element into the phasor domain:

$V_1 = 20\angle -36.87°$ V $\quad V_2 = 50\angle -106.26°$

$Z_L = j2000 \times 1 \times 10^{-3} = j2\,\Omega$

$Z_c = \dfrac{1}{j2000 \times 100 \times 10^{-6}} = \dfrac{1}{j0.2} = -j5\,\Omega$

So the equivalent circuit in the phasor domain is:



The MCM equations are:

$$j2I_1 + 10(I_1 - I_2) = 20\angle{-36.87^\circ}$$

$$-j5I_2 + 10(I_2 - I_1) = -50\angle{-106.26^\circ}.$$

Put in canonical form:

$$(10+j2)I_1 - 10I_2 = 20\angle{-36.87} \qquad (1)$$

$$-10I_1 + (10-j5)I_2 = -50\angle{-106.26^\circ} \qquad (2)$$

Divide (1) by $(10+j2)$ and (2) by 10 and eliminate I_1 by adding the two equations:

$$\frac{-10}{10+j2} I_2 + \frac{10-j5}{10} I_2 = \frac{20\angle{-36.87}}{10+j2} - \frac{50\angle{-106.26^\circ}}{10}$$

$$I_2(0.03846 - j0.3077) = 2.70767 + j3.3385$$

$$I_2 = -9.6 + j10$$

From Equation (1):

$$I_1 = (20\angle{-36.87} + 10(-9.6+j10))/(10+j2)$$

$$= (-80+j88)/(10+j2) = -6+j10 \text{ A}$$

$$V_o = (I_1 - I_2)\times 10 = (-6+j10+9.6-j10)\times 10$$

$$= 36\angle 0$$

$$v_o(t) = 36\cos(2000t)$$

$$(50+j70)I_1 - 640 I_2 = 72$$

$$-50 I_1 + (800 - j400) I_2 = 0$$

Eliminate I_1 :

$$-\frac{640}{50+j70} I_2 + \frac{800-j400}{50} I_2 = \frac{72}{50+j70} \Rightarrow$$

$$I_2 (11.676 - j1.946) = \frac{72}{50+j70} \Rightarrow$$

$$I_2 = 0.05 - j0.05 \text{ A}$$

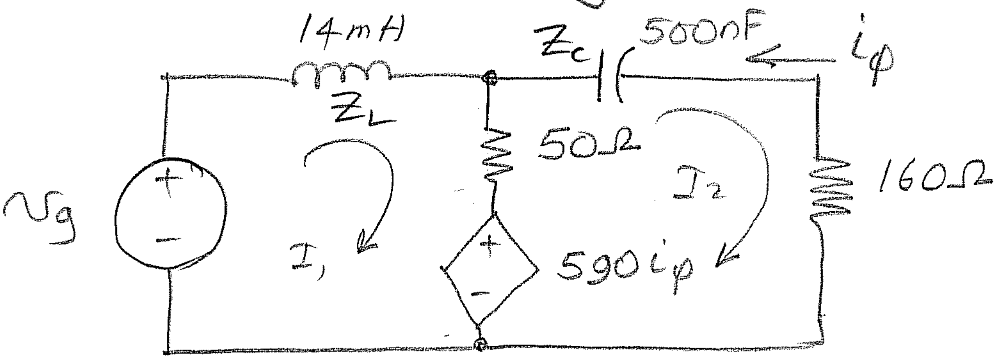
$$V_o = I_2 \times 160 = 8 - j8 = 11.31\angle -45^\circ$$

So $v_o(t) = 11.31 \cos(5000t - 45^\circ)$ V

Solve also Problems 9.61, 9.62

Problem 9.64

Use the mesh current method to find the steady state expression for v_o in the following circuit. Note that $v_g = 72 \cos(5000t)$ V



$$Z_L = 14\times10^{-3}\times 5000 = j\,70\,\Omega$$

$$Z_c = \frac{1}{j5000\times 500\times10^{-9}} = -j400\,\Omega$$

$$V_g = 72\angle 0 \text{ V}$$

The MCM equations are:

$$j70 I_1 + (I_1 - I_2)50 + 590 I_\varphi = 72\angle 0$$

$$-j400 I_2 + 160 I_2 - 590 I_\varphi + (I_2 - I_1)50 = 0$$

with $I_\varphi = -I_2$